

Math 150 Hand-In Assignment 1

The following questions are divided into two parts. All students should work on the suggested practice problems. The hand-in part is not mandatory. Its purpose is to identify and train the best and the most motivated students in the class and to help them achieve a deeper level of understanding of calculus. Some questions may be very hard and the student should not be discouraged. In mathematics one often battles with a problem for weeks without success, but this battle slowly makes one more durable and stronger mathematician.

Suggested Practice Problems

1. Find the limit or explain why the limit doesn't exist.

(a) $\lim_{x \rightarrow 0} \frac{x^2 + x}{\sqrt{x^3 + x^2}}$

(b) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

(c) $\lim_{x \rightarrow -3} \frac{x+2}{x+3}$

(d) $\lim_{x \rightarrow 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$

(e) $\lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x - 5}$

(f) $\lim_{t \rightarrow 1} \frac{t^4 - 1}{t^3 - 1}$

(g) $\lim_{h \rightarrow 0} \frac{(x+h)^7 - x^7}{h}$ [Hint: Notice that limit is taken with respect to h.

Think of x as fixed]

(h) $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

(i) $\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

Problems to Hand-In

2. Derive a factorization formula for $x^5 + a^5$. [Hint: Reduce the problem to the one you know how to solve. Specifically, write $x^5 + a^5 = x^5 - b^5$ for an

appropriately chosen b , factor the latter expression, and finally express everything back in terms of a .]

3. If $n = 2k + 1$ (i.e. n is odd), express $x^n + a^n$ as the product of $x + a$ and some other polynomial.
4. Find the limit $\lim_{x \rightarrow 1} \frac{x^3 - 1}{\sqrt{x} - 1}$. [Hint: How would you factor $\sqrt{x} - 1$ from $x^3 - 1$?
This is easier than it looks!]